FORM TP 2019090

TEST CODE **01234020**

MAY/JUNE 2019

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO sections: I and II.
2. Section I has SEVEN questions and Section II has THREE questions.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. All working MUST be clearly shown.
7. **A list of formulae is provided on page 4 of this booklet.**
8. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
9. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Required Examination Materials

Electronic calculator
Geometry set

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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LIST OF FORMULAE

Volume of a prism — $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of a cylinder — $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid — $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference — $C = 2\pi r$ where r is the radius of the circle.

Arc length — $S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.

Area of a circle — $A = \pi r^2$ where r is the radius of the circle.

Area of a sector — $A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.

Area of a trapezium — $A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations — If $ax^2 + bx + c = 0$,

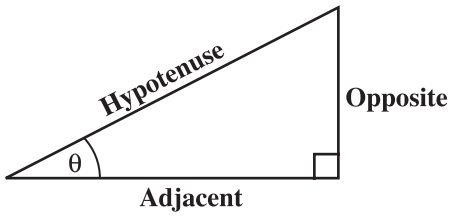
$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric ratios

$$\sin\theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$\cos\theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

$$\tan\theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$



Area of a triangle — Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

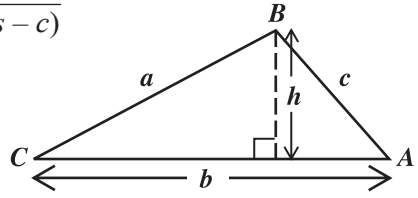
$$\text{Area of } \Delta ABC = \frac{1}{2}ab\sin C$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Cosine rule — $a^2 = b^2 + c^2 - 2bc\cos A$

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SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Using a calculator or otherwise, evaluate EACH of the following:

(i) $\dfrac{2\frac{1}{4} - 1\frac{3}{5}}{3}$

.....

(2 marks)

(ii) 2.14 sin 75°, giving your answer to 2 decimal places

.....

(1 mark)

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(b) Irma's take-home pay is $4 320 per fortnight (every two weeks). Each fortnight Irma's pay is allocated according to the following table.

Item	Amount Allocated
Rent	$\$x$
Food	\$629
Other living expenses	$\$2x$
Savings	\$1 750
Total	**\$4 320**

(i) What is Irma's **annual** take-home pay? (Assume she works 52 weeks in any given year.)

.....

(1 mark)

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(ii) Determine the amount of money that Irma allocates for rent each month.

.....

(3 marks)

(iii) All of Irma's savings is used to pay her son's university tuition cost, which is $150 000.

If Irma's pay remains the same and she saves the same amount each month, what is the MINIMUM number of years that she must work in order to save enough money to cover her son's tuition cost?

.....

(2 marks)

Total 9 marks

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2. (a) Simplify completely

(i) $3p^2 \times 4p^5$

.....

(1 mark)

(ii) $\frac{3x}{4y^3} \div \frac{21x^2}{20y^2}$.

.....

(2 marks)

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(b) Solve the equation

$$\frac{3}{7x-1} + \frac{1}{x} = 0.$$

.....

(3 marks)

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(c) When a number, x, is multiplied by 2, the result is squared to give a new number, y.

(i) Express y in terms of x.

.....

(1 mark)

(ii) Determine the two values of x that satisfy the equation $y = x$ AND the equation derived in (c) (i).

.....

(2 marks)

Total 9 marks

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3. (a) Using a ruler, a pencil and a pair of compasses only, construct the triangle *NLM*, in which $LM = 12$ cm, $\angle MLN = 30°$ and $\angle LMN = 90°$.

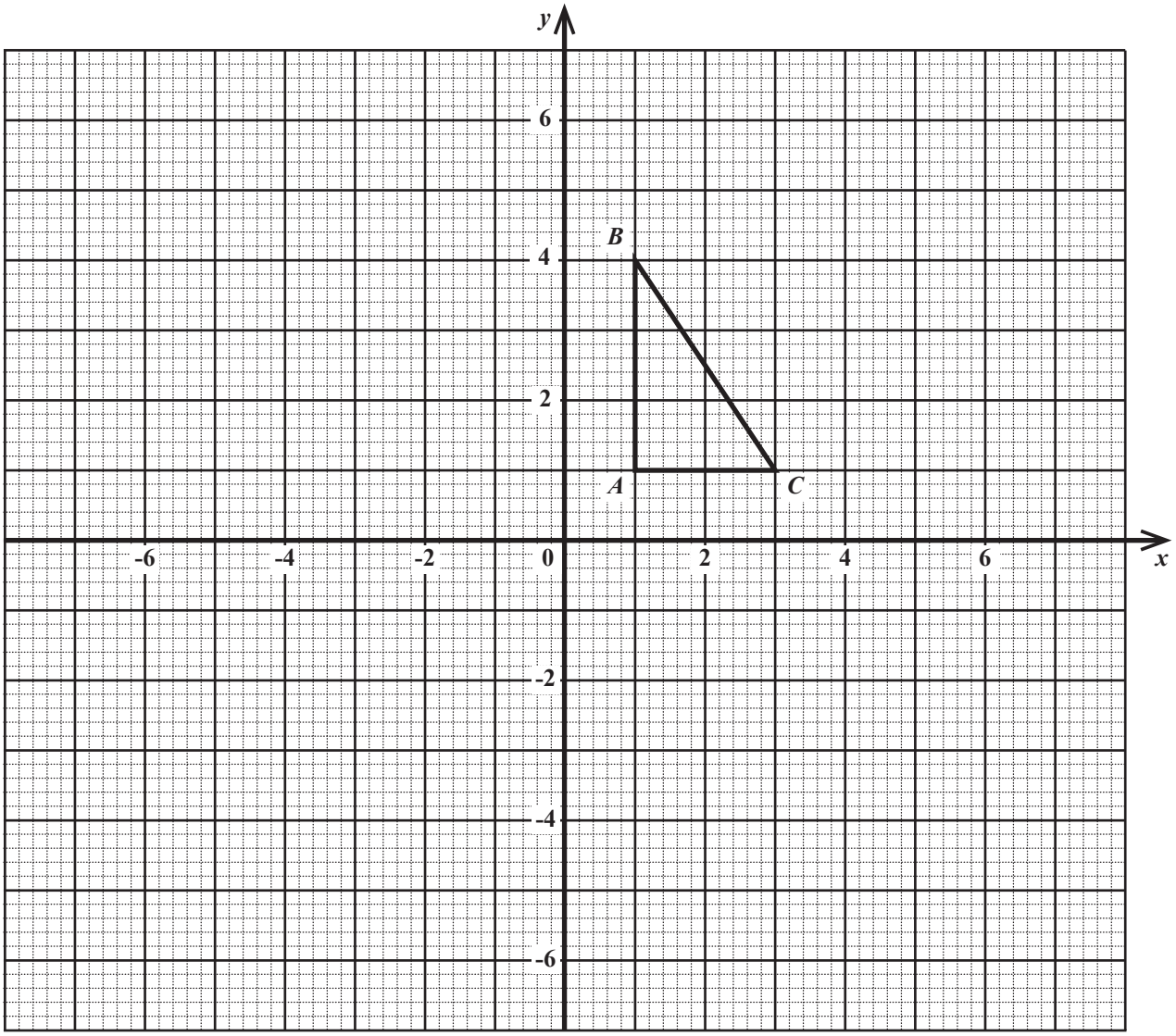
(Credit will be given for clearly drawn construction lines.)

(4 marks)

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(b) Triangle ABC with vertices A (1, 1) , B (1, 4) and C (3, 1) is shown on the diagram below.



ΔABC is mapped onto ΔLMN by a reflection in the x-axis followed by a reflection in the y-axis.

(i) On the diagram, draw and label ΔLMN. **(2 marks)**

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(ii) Describe fully a single transformation that maps ΔABC onto ΔLMN.

.....

.....

.....

(2 marks)

(iii) State the 2×2 matrix for the transformation that maps ΔABC onto ΔLMN.

.....

(1 mark)

Total 9 marks

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4. (a) The quantity P varies inversely as the square of V.

(i) Using the letters P, V and k, form an **equation** connecting the quantities P and V.

.....

(1 mark)

(ii) Given that $V = 3$ when $P = 4$, determine the positive value of V when $P = 1$.

.....

(2 marks)

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(b) (i) Given that x is a real number, solve the inequality

$$-7 < 3x + 5 \leq 7.$$

.....

(2 marks)

(ii) Represent your answer in (b) (i) on the number line shown below.



(1 mark)

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(c) The equation of a straight line is given as

$$\frac{x}{3} + \frac{y}{7} = 1.$$

This line crosses the y-axis at Q.

(i) Determine the coordinates of Q.

.....

(1 mark)

(ii) What is the gradient of this line?

.....

(2 marks)

Total 9 marks

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5. The cumulative frequency distribution of the volume of petrol needed to fill the tanks of 150 different vehicles is shown below.

Volume (litres)	Cumulative Frequency
11–20	24
21–30	59
31–40	101
41–50	129
51–60	150

(a) For the class 21–30, determine the

(i) lower class boundary

.....

(1 mark)

(ii) class width.

.....

(1 mark)

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(b) How many vehicles were recorded in the class 31–40?

.....

(1 mark)

(c) A vehicle is chosen at random from the 150 vehicles. What is the probability that the volume of petrol needed to fill its tank is **more** than 50.5 litres? **Leave your answer as a fraction.**

.....

(2 marks)

(d) Byron estimates the median amount of petrol to be 43.5 litres. Explain why Byron's estimate is INCORRECT.

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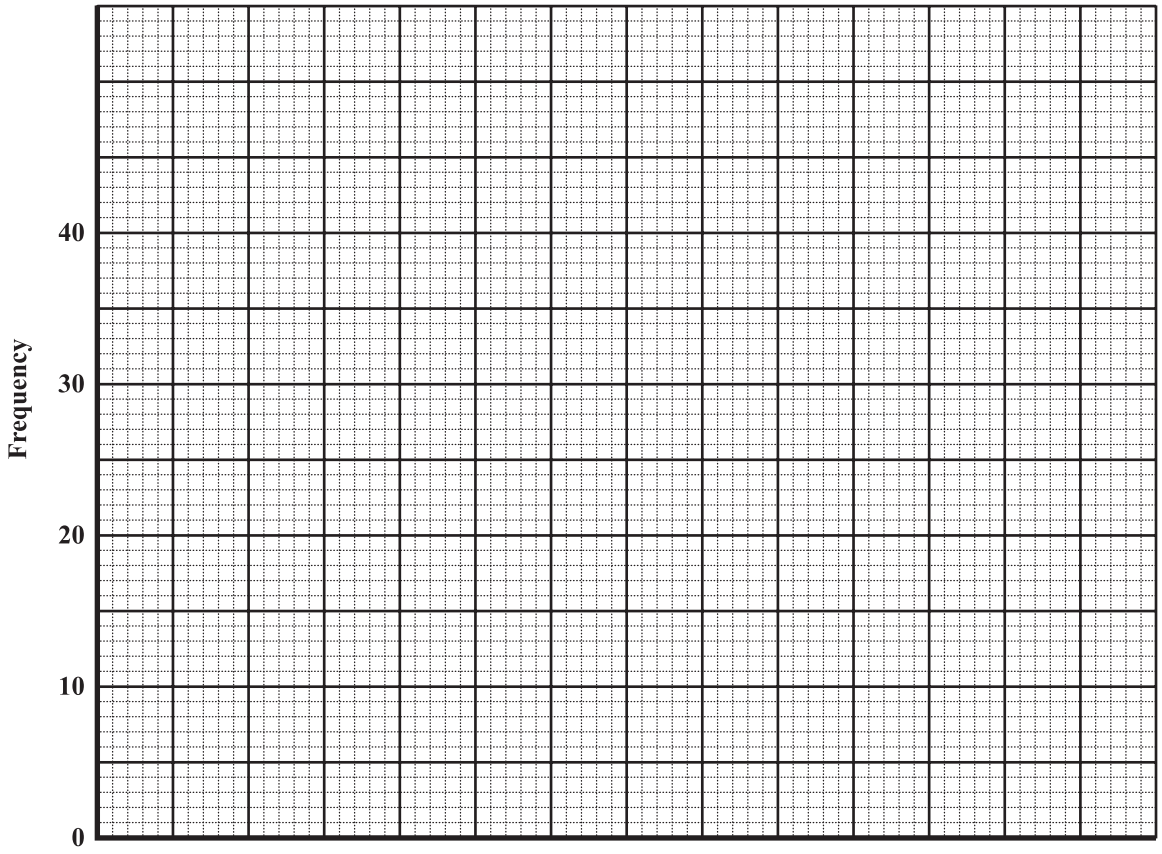
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(e) On the partially labelled grid below, construct a histogram to represent the distribution of the volume of petrol needed to fill the tanks of the 150 vehicles.



(3 marks)

Total 9 marks

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6. (a) The scale on a map is 1:25 000.

(i) Determine the actual distance, in km, represented by 0.5 cm on the map.

.....

(2 marks)

(ii) Calculate the actual area, in km^2, represented by 2.25 cm^2 on the map.

.....

(3 marks)

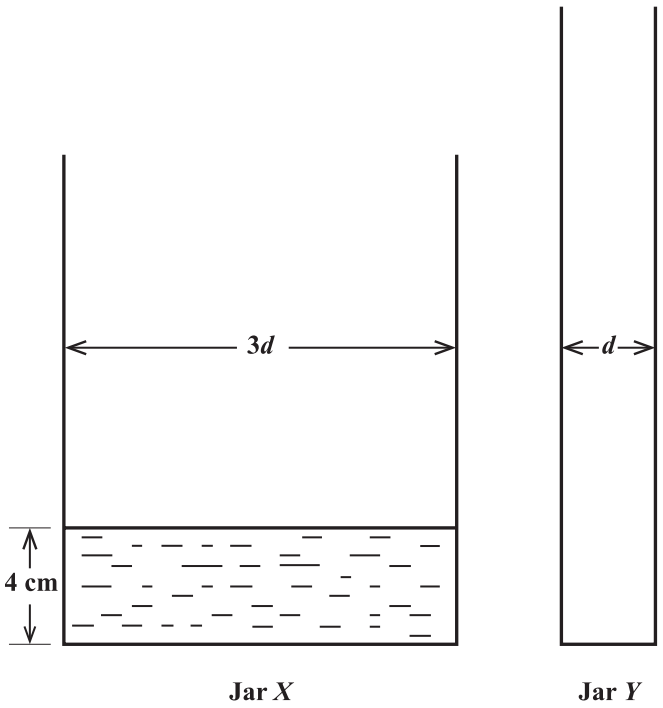
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(b) The diagram below **(not drawn to scale)** shows the cross-section of two cylindrical jars, Jar *X* and Jar *Y*. The diameters of Jar *X* and Jar *Y* are 3*d* cm and *d* cm respectively.

Initially, Jar *Y* is empty and Jar *X* contains water to a height (depth) of 4 cm.



(i) Determine, in terms of π and *d*, the volume of water in Jar *X*.

.....

(2 marks)

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(ii) If all the water from Jar *X* is now poured into Jar *Y*, calculate the height it will reach.

.....

(2 marks)

Total 9 marks

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7. (a) The nth term, T_n, of a sequence is given by

$$T_n = 3n^2 - 2.$$

(i) Show that the first term of the sequence is 1.

.....

(1 mark)

(ii) What is the third term of the sequence?

.....

(1 mark)

(iii) Given that $T_n = 145$, what is the value of n?

.....

(3 marks)

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(b) The first 8 terms of another sequence with n^{th} term, $U(n)$, are

1, 1, 2, 3, 5, 8, 13, 21

where

$U(1) = 1$
$U(2) = 1$ and
$U(n) = U(n-1) + U(n-2)$ for $n \geq 3$.

For example, the fifth and seventh terms are

$U(5) = U(4) + U(3) = 3 + 2 = 5$
$U(7) = U(6) + U(5) = 8 + 5 = 13$.

(i) Write down the next two terms in the sequence, that is, $U(9)$ and $U(10)$.

.....

(2 marks)

(ii) Which term in the sequence is the sum of $U(18)$ and $U(19)$?

.....

(1 mark)

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(iii) Show that $U(20) - U(19) = U(19) - U(17)$.

.....

(2 marks)

Total 10 marks

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SECTION II

Answer ALL questions.

All working must be clearly shown.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The functions f and g are defined by

$$f(x) = \frac{9}{2x + 1} \quad \text{and} \quad g(x) = x - 3.$$

(i) State a value of x that CANNOT be in the domain of f.

.....

(1 mark)

(ii) Find, in its simplest form, expressions for

a) $fg(x)$

.....

(2 marks)

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b) $f^{-1}(x)$.

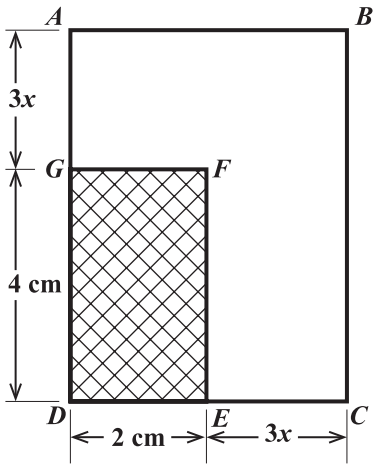
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(2 marks)

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(b) The diagram below shows two rectangles, *ABCD* and *GFED*. *ABCD* has an area of 44 cm^2. *GFED* has sides 4 cm and 2 cm. $AG = EC = 3x$ cm.



(i) By writing an expression for the area of rectangle *ABCD*, show that $x^2 + 2x - 4 = 0$.

.....

(3 marks)

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(ii) Calculate, to 3 decimal places, the value of x.

.....

(2 marks)

(iii) Calculate the perimeter of the UNSHADED region.

.....

(2 marks)

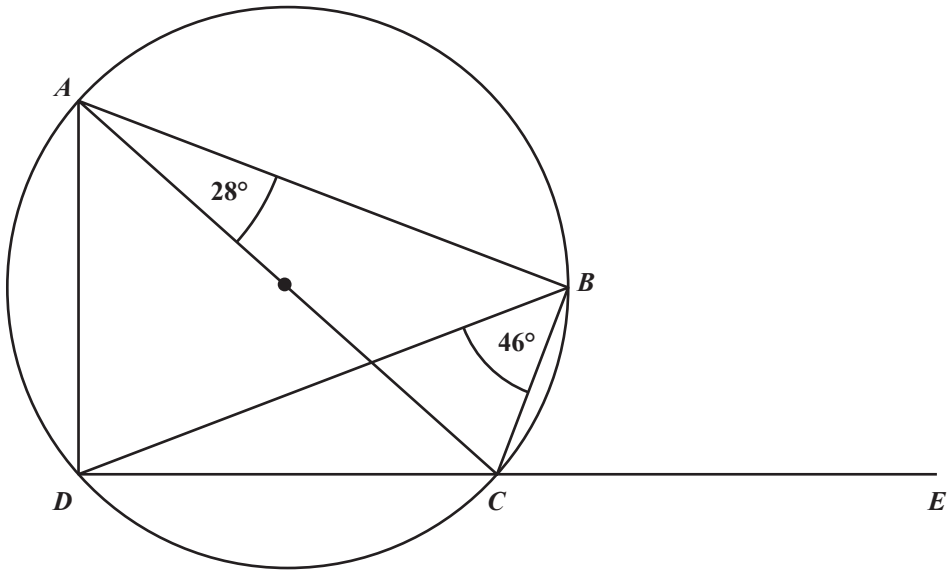
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GEOMETRY AND TRIGONOMETRY

9. (a) The diagram below shows a circle where AC is a diameter. B and D are two other points on the circle and DCE is a straight line. Angle $CAB = 28°$ and $\angle DBC = 46°$.



Calculate the value of each of the following angles. Show detailed working where necessary and give a reason to support your answers.

(i) $\angle DBA$

Reason .....

.....

.....

(2 marks)

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(ii) $\angle DAC$

Reason .....

.....

.....

(2 marks)

(iii) $\angle BCE$

Reason .....

.....

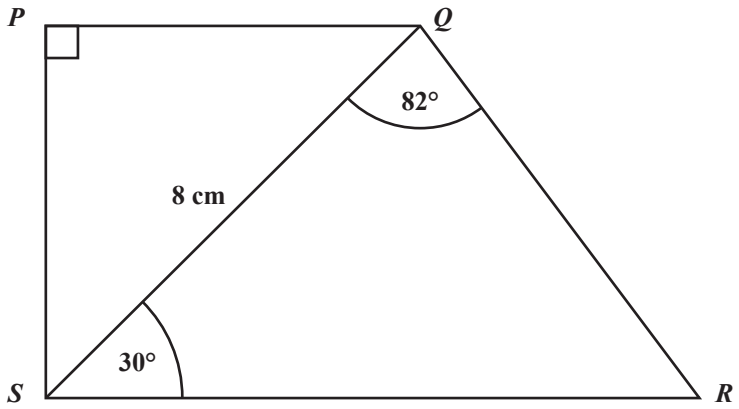
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(b) The diagram below shows a quadrilateral $PQRS$ where PQ and SR are parallel, $SQ = 8$ cm, $\angle SPQ = 90°$, $\angle SQR = 82°$ and $\angle QSR = 30°$.



Determine

(i) the length PS

.....

(2 marks)

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(ii) the length PQ

.....

(1 mark)

(iii) the area of $PQRS$.

.....

(3 marks)

Total 12 marks

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VECTORS AND MATRICES

10. (a) (i) a) Find the matrix product

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix} \begin{pmatrix} k \\ 5 \end{pmatrix}.$$

.....

(2 marks)

b) Hence, find the values of h and k that satisfy the matrix equation

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix} \begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

.....

(2 marks)

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(ii) Using a matrix method, solve the simultaneous equations

$2x + 3y = 5$
$-5x + y = 13.$

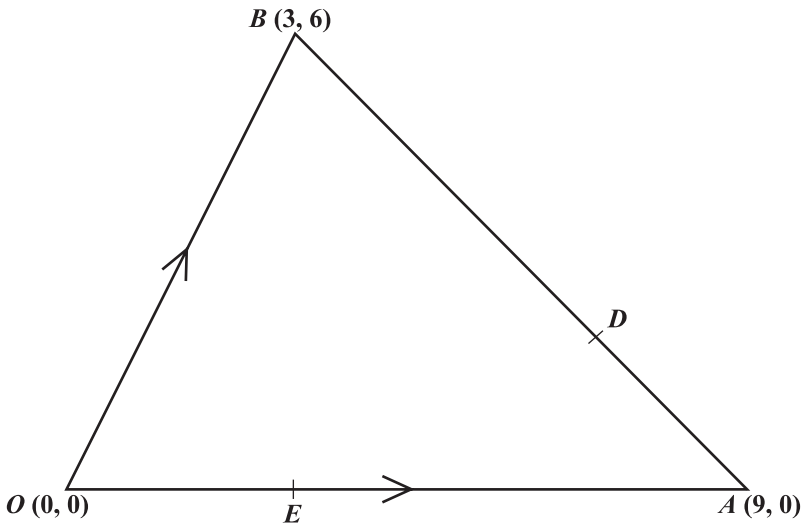
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(b) Relative to the origin O (0, 0), the position vectors of the points A and B are $OA = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$ and $OB = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ respectively. The points D and E are on AB and OA respectively and are such that $AD = \frac{1}{3} AB$ and $OE = \frac{1}{3} OA$. The following diagram illustrates this information.



Express the following vectors in the form $\begin{pmatrix} a \\ b \end{pmatrix}$.

(i) $\overrightarrow{AB}$

.....

(1 mark)

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(ii) $\overrightarrow{OD}$

.....

(2 marks)

(iii) $\overrightarrow{BE}$

.....

(2 marks)

Total 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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