FORM TP 2024046

TEST CODE 01254020

MAY/JUNE 2024

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

061

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of SIX questions. Answer ALL questions.
2. Write your answers in the spaces provided in this booklet.
3. Do NOT write in the margins.
4. A list of formulae is provided on page 4 of this booklet.
5. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
6. **If you use the extra page(s), you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

A000

Required Examination Materials

Electronic calculator (non-programmable)
Geometry set

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

Copyright © 2023 Caribbean Examinations Council
All rights reserved.

LIST OF FORMULAE

Arithmetic series $T_n = a + (n-1)d \qquad S_2 = \frac{n}{2}[2a + (n-1)d]$

Geometric series $T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \qquad S_\infty = \frac{a}{1-r}, -1 < r < 1 \text{ or } |r| < 1$

Circle $x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x+f)^2 + (y+g)^2 = r^2$

Vectors $\mathbf{v} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{a}| \bullet |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)} \text{ where } \mathbf{v} = x\mathbf{i} + y\mathbf{j}$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Kinematics $v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2 \qquad v = \frac{dx}{dt} = \dot{x}$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \ddot{x}$$

Calculus $\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1} \qquad \frac{d}{dx}\sin ax = a\cos ax$

$$\frac{d}{dx}\cos ax = -a\sin ax \qquad s = \int v\,dt \qquad v = \int a\,dt$$

Statistics $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$

Probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

DO NOT WRITE IN THIS AREA

061

A000

SECTION I

ALGEBRA, SEQUENCES AND SERIES

ALL working must be clearly shown.

1. (a) (i) Determine the other linear factors of the polynomial $3x^3 + 8x^2 - 20x - 16$, given that $x - 2$ is a factor.

(3 marks)

(ii) Hence, simplify the multiplication

$$\frac{3x^3 + 8x^2 - 20x - 16}{x^2 - 4} \times \frac{x+2}{x+4}.$$

(2 marks)

(b) The equation $kx^2 + x - 15 = 10$ has roots α and β, where $k \in W$.

(i) Determine expressions for

- $\alpha + \beta$
- $\alpha\beta$.

(2 marks)

(ii) Given that $\alpha^2 + \beta^2 = \frac{61}{4}$, use the expressions in (b) (i) to determine the value of k.

(4 marks)

DO NOT WRITE IN THIS AREA

(c) For the quadratic function $g(x) = -5x^2 - 4x + 2$, determine the exact value of the maximum point **and** the range using the method of completing the square, or otherwise.

(4 marks)

Total 15 marks

2. (a) Write the expression $2log_3 x + 2 - log_3 y$ as a single term.

(3 marks)

(b) (i) By using logarithms, express the relationship $V = 7 \times 5^t$ in linear form.

(2 marks)

(ii) Hence, state the value of the gradient of the line which represents the relationship in (b) (i).

(1 mark)

(c) Rationalize the denominator of the expression $\frac{1+\sqrt{2}}{3-\sqrt{2}}$.

(4 marks)

(d) Evaluate $\sum_{t=0}^{4} 5^{t-2}$.

(3 marks)

(e) Determine whether the following sequence is divergent or convergent. Justify your response.

$$1, \quad -\frac{2}{3}, \quad \frac{4}{9}, \quad -\frac{8}{27}, \ldots$$

(2 marks)

Total 15 marks

DO NOT WRITE IN THIS AREA

SECTION II

COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY

ALL working must be clearly shown.

3. (a) Determine the points of intersection of the circle $x^2 + y^2 - 4x + 6y + 8 = 0$ and the line $y = x - 6$.

(5 marks)

(b) (i) Given that the coordinates of the points A and B are $(7, -3)$ and $(2, 1)$ respectively, state the position vectors corresponding to the points A and B in the form $x\mathbf{i} + y\mathbf{j}$.

(1 mark)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(ii) Determine $\overrightarrow{AB}$.

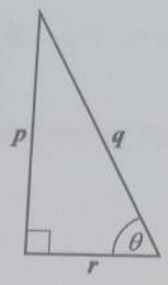


(2 marks)

(iii) Calculate the value of the scalar product $\boldsymbol{OA.OB}$.

(2 marks)

(c) (i) Using the letters p, q or r, write an expression for EACH of the following trigonometric ratios.



$\sin \theta =$

$\cos \theta =$

(2 marks)

DO NOT WRITE IN THIS AREA

(ii) Using your answers in (c) (i), determine the value of $\sin^2\theta + \cos^2\theta$.
Recall that $\sin^2\theta = (\sin\theta)^2$

(4 marks)

(d) Prove the identity $\tan A + \frac{1}{\tan A} = \frac{1}{\sin A \cos A}$.

(4 marks)

Total 20 marks

SECTION III

INTRODUCTORY CALCULUS

ALL working must be clearly shown.

4. (a) (i) Use the definition of the derivative as a limit to find $f'(x)$ for the function $f(x) = x^2 - 3$.

(4 marks)

(ii) Hence, determine the value of $f''(5)$.

(2 marks)

GO ON TO THE NEXT PAGE

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(b) Given that $y = \frac{\sin x}{\cos x}$, show that $\frac{dy}{dx} = \frac{1}{\cos^2 x}$.

(4 marks)

(c) The equation of a curve is given by $y = (5x^2 - 7)^4$. Determine the equation of the gradient of the curve.

(5 marks)

Total 15 marks

5. (a) The region enclosed between the curve $y = x^2 + 1$, the x-axis, the y-axis and the line $x = 2$ is rotated about the x-axis through an angle of 360°.

Calculate the volume of the solid of revolution that is formed.

(5 marks)

(b) As a particle moves along a straight line, its displacement is measured from a fixed point, O, on the line. At time t seconds, the acceleration, a, is given by $a = 24t - 14$.

(i) Find the expression for the velocity, v, of the particle, given that when $t = 1$, $v = 3$ ms^{-1}.

(3 marks)

(ii) Using your answer in (b) (i), find an expression for the displacement, s, of the particle, given that when $t = 1$, $s = 10$ m.

(3 marks)

(c) The gradient function of the curve C is given by $7 - 2x$. If the curve passes through the point (3, 8), find the equation of the curve.

(4 marks)

Total 15 marks

SECTION IV

PROBABILITY AND STATISTICS

ALL working must be clearly shown.

6. (a) The heights of 35 children at a nursery were recorded to the nearest centimetre. The data is shown below.

61	118	79	90	83	70	80
95	75	76	92	62	115	79
71	103	109	84	65	86	92
111	78	94	74	99	81	108
108	67	109	92	79	116	62

(i) Display the data shown above on a stem-and-leaf diagram.

(3 marks)

(ii) From the stem-and-leaf diagram in (a) (i), determine the value of the

- median

- lower quartile

- upper quartile.

(3 marks)

(iii) At another nearby nursery, the heights of children of the same age were recorded. The median height was 79 cm and the interquartile range was 24 cm.

Compare the characteristics of the two groups of children at the two nurseries and describe ONE distinct observation about the distributions.

.....

.....

.....

.....

.....

.....

(2 marks)

(b) (i) A bag contains 6 red marbles and 5 black marbles. During 2 rounds of a marble game, a student is required to randomly draw 1 marble without replacement for each round. Construct a probability tree diagram to represent this information.

(4 marks)

(ii) Using your answer in (b) (i), find the probability that the marbles drawn are ALL the SAME colour.

(4 marks)

(c) Given two events, A and B, $P(A \cup B) = 0.7$, $P(A) = 0.3$ and $P(B') = 0.6$.

(i) Calculate $P(A \cap B)$.

(3 marks)

(ii) What is the relationship between Events A and B?

(1 mark)

Total 20 marks