FORM TP 2014037

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C A R I B B E A N E X A M I N A T I O N S C O U N C I L

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

06 MAY 2014 (p.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of FOUR sections. Answer ALL questions in Section 1, Section 2 and Section 3.
2. Answer ONE question in Section 4.
3. Write your solutions with full working in the booklet provided.
4. A list of formulae is provided on page 2 of this booklet.

Required Examination Materials

Electronic Calculator (non programmable)
Geometry Set
Mathematical Tables (provided)
Graph Paper (provided)

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LIST OF FORMULAE

Arithmetic Series $T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric Series $T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r},\ -1 < r < 1 \text{ or } |r| < 1$

Circle $x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x + f)^2 + (y + g)^2 = r^2$

Vectors $\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)} \text{ where } \mathbf{v} = x\mathbf{i} + y\mathbf{j}$

Trigonometry $\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Differentiation $\frac{\mathrm{d}}{\mathrm{d}x}(ax + b)^n = an(ax + b)^{n-1}$

$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$

$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$

Statistics $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$

Probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Kinematics $v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$

GO ON TO THE NEXT PAGE

SECTION 1

Answer BOTH questions.

ALL working must be clearly shown.

1. (a) (i) The function f is defined by $f : x \rightarrow 1 - x^2$, $x \in \mathbb{R}$.

Show that f is NOT one-to-one. **(1 mark)**

(ii) The function g is defined by $g : x \rightarrow \frac{1}{2}x - 3$, $x \in \mathbb{R}$.

a) Find $fg(x)$, and clearly state its domain. **(2 marks)**

b) Determine the inverse, g^{-1}, of g and sketch on the same pair of axes, the graphs of g and g^{-1}. **(3 marks)**

(b) When the expression $2x^3 + ax^2 - 5x - 2$ is divided by $2x - 1$, the remainder is -3.5.

Determine the value of the constant a. **(3 marks)**

(c) The length of a regtangular kitchen is y m and the width is x m. If the length of the kitchen is half the square of its width and its perimeter is 48 m, find the values of x and y (the dimensions of the kitchen). **(5 marks)**

Total 14 marks

2. (a) Given that $f(x) = -2x^2 - 12x - 9$.

(i) Express $f(x)$ in the form $k + a(x + h)^2$, where a, h and k are integers to be determined. **(3 marks)**

(ii) State the maximum value of $f(x)$. **(1 mark)**

(iii) Determine the value of x for which $f(x)$ is a maximum. **(1 mark)**

(b) Find the set of values of x for which $3 + 5x - 2x^2 \leq 0$. **(4 marks)**

(c) A series is given by $0.2 + 0.02 + 0.002 + 0.0002 + ...$

(i) Show that this series is geometric. **(3 marks)**

(ii) Find the sum to infinity of this series, giving your answer as an exact fraction. **(2 marks)**

Total 14 marks

SECTION 2

Answer BOTH questions.

ALL working must be clearly shown.

3. (a) (i) Determine the value of k such that the lines $x + 3y = 6$ and $kx + 2y = 12$ are perpendicular to each other. **(3 marks)**

(ii) A circle of radius 5 cm has as its centre the point of intersection of the two perpendicular lines in (i). Determine the equation for this circle. **(3 marks)**

(b) RST is a triangle in the coordinate plane. Position vectors R, S, and T relative to an origin, O, are $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ respectively.

(i) Show that $T\hat{R}S = 90°$. **(4 marks)**

(ii) Determine the length of the hypotenuse. **(2 marks)**

[Hint: A rough drawing of RST might help].

Total 12 marks

4. (a) Figure 1 shows the sector *OAB* of a circle with centre *O*, radius 9 cm and angle 0.7 radians.

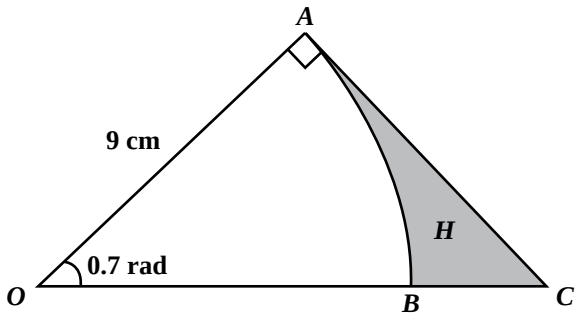


Figure 1.

(i) Find the area of the sector *OAB*. **(2 marks)**

(ii) Hence, find the area of the shaded region, *H*. **(4 marks)**

(b) Given that $\sin \frac{\pi}{6} = \frac{1}{2}$ and $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, show that

$$\cos\left(x + \frac{\pi}{6}\right) = \frac{1}{2}\left(\sqrt{3}\cos x - \sin x\right), \text{ where } x \text{ is acute.}$$ **(2 marks)**

(c) Prove the identity $\left(\frac{\tan\theta \sin\theta}{1 - \cos\theta}\right) \equiv 1 + \frac{1}{\cos\theta}$. **(4 marks)**

Total 12 marks

SECTION 3

Answer BOTH questions.

ALL working must be clearly shown.

5. (a) The equation of a curve is $y = 3 + 4x - x^2$. The point P (3, 6) lies on the curve.

Find the equation of the tangent to the curve at P, giving your answer in the form

$ax + by + c = 0$, where $a, b, c, \in \mathbb{Z}$. **(4 marks)**

(b) Given that $f(x) = 2x^3 - 9x^2 - 24x + 7$.

(i) Find ALL the stationary points of $f(x)$. **(5 marks)**

(ii) Determine the nature of EACH of the stationary points of $f(x)$. **(5 marks)**

Total 14 marks

6. (a) Evaluate $\int_2^4 x\,(x^2 - 2)\,dx$. **(4 marks)**

(b) Evaluate $\int_0^{\frac{\pi}{3}} (4\cos x + 2\sin x)\,dx$, leaving your answer in surd form. **(4 marks)**

(c) A curve passes through the point P (2, –5) and is such that $\frac{dy}{dx} = 6x^2 - 1$.

(i) Determine the equation of the curve. **(3 marks)**

(ii) Find the area of the finite region bounded by the curve, the x-axis, the line $x = 3$ and the line $x = 4$. **(3 marks)**

Total 14 marks

SECTION 4

Answer ONLY ONE question.

ALL working must be clearly shown.

7. (a) There are 60 students in the sixth form of a certain school. Mathematics is studied by 27 of them, Biology by 20 of them and 22 students study neither Mathematics nor Biology. If a student is selected at random, what is the probability that the student is studying

(i) both Mathematics and Biology? **(3 marks)**

(ii) Biology only? **(2 marks)**

(b) Two ordinary six-sided dice are thrown together. The random variable S represents the sum of the scores of their faces landing uppermost.

(i) Copy and complete the sample space diagram below.

6			9			
5		7				
4						10
3					8	
2				6		
1	2					
	1	**2**	**3**	**4**	**5**	**6**

Sample space diagram of S

(1 mark)

(ii) Find

a) $P(S > 9)$ **(2 marks)**

b) $P(S \leq 4)$. **(1 mark)**

(iii) Let D be the difference between the scores of the faces landing uppermost. The table below gives the probability of each possible value of d.

d	0	1	2	3	4	5
$P(D = d)$	$\frac{1}{6}$	a	$\frac{2}{9}$	b	$\frac{1}{9}$	c

Find the values of a, b and c. **(3 marks)**

(c) The aptitude scores obtained by 51 applicants for a supervisory job are summarized in the following stem and leaf diagram.

5|1 means 51

3	1 5 9
4	2 4 6 8 9
5	1 3 3 5 6 7 9
6	0 1 3 3 3 5 6 8 8 9
7	1 2 2 2 4 5 5 5 6 8 8 8 9 9
8	0 1 2 3 5 8 8 9
9	0 1 2 6

(i) Find the median and quartiles for the data given. **(4 marks)**

(ii) Construct a box-and-whisker plot to illustrate the data given and comment on the distribution of the data. **(4 marks)**

Total 20 marks

8. (a) Figure 2 below, **not drawn to scale**, shows the motion of a car with velocity, *V*, as it moves along a straight road from Point *A* to Point *B*. The time, *t*, taken to travel from Point *A* to Point *B* is 90 seconds and the distance from Point *A* to Point *B* is 1410 m.

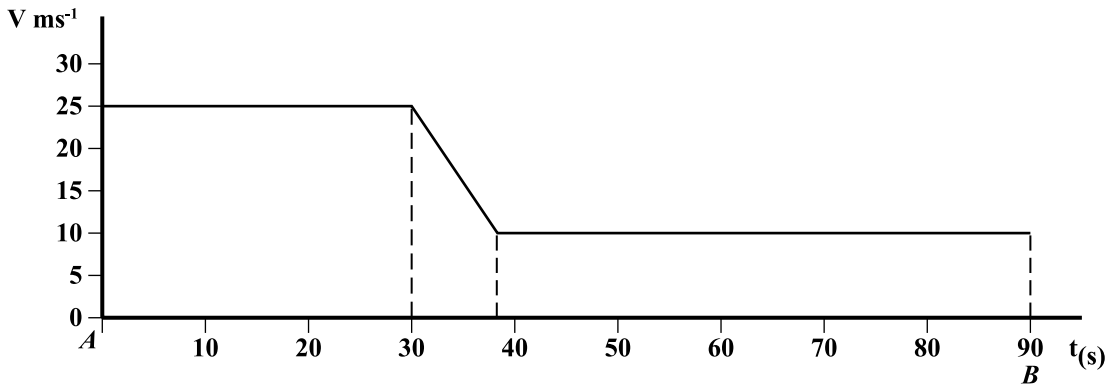


Figure 2.

(i) What distance did the car travel from Point *A* towards Point *B* before starting to decelerate? **(2 marks)**

(ii) Calculate the deceleration of the car as it goes from 25 m s^{-1} to 10 m s^{-1}. **(5 marks)**

(iii) For how long did the car maintain the speed of 10 m s^{-1}? **(1 mark)**

(iv) From Point *B*, the car decelerates uniformly, coming to rest at a Point *C* and covering a further distance of 30 m. Determine the average velocity of the car over the journey from Point *A* to Point *C*. **(2 marks)**

(b) A particle travels along a straight line. It starts from rest at a point, P, on the line and after 10 seconds, it comes to rest at another point, Q, on the line. The velocity v m s^{-1} at time t seconds after leaving P is

$$v = 0.72t^2 - 0.096t^3 \qquad \text{for } 0 \leq t \leq 5$$

$$v = 2.4t - 0.24t^2 \qquad \text{for } 5 \leq t \leq 10$$

At maximum velocity the particle has no acceleration.

(i) Find the time when the velocity is at its maximum. **(3 marks)**

(ii) Determine the maximum velocity. **(2 marks)**

(iii) Find the distance moved by the particle from P to the point where the particle attains its maximum velocity. **(5 marks)**

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.