TEST CODE **01254020**

FORM TP 2021042

MAY/JUNE 2021

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of SIX questions. Answer ALL questions.
2. Write your answers in the spaces provided in this booklet.
3. Do NOT write in the margins.
4. A list of formulae is provided on page 4 of this booklet.
5. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
6. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Required Examination Materials

Electronic calculator (non-programmable)
Geometry set

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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LIST OF FORMULAE

Arithmetic Series $T_n = a + (n-1)d \qquad S_2 = \frac{n}{2}\left[2a + (n-1)d\right]$

Geometric Series $T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \qquad S_\infty = \frac{a}{1-r}, -1 < r < 1 \text{ or } |r| < 1$

Circle $x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x+f)^2 + (y+g)^2 = r^2$

Vectors $\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)}$ where $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Differentiation $\frac{\mathrm{d}}{\mathrm{d}x}(ax+b)^n = an(ax+b)^{n-1} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$

Statistics $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$

Probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Kinematics $v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2 \qquad v = \frac{dx}{dt} = \dot{x}$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \ddot{x}$$

Calculus $\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1} \qquad \frac{d}{dx}\sin ax = a\cos ax$

$\frac{d}{dx}\cos ax = -a\sin ax \qquad s = \int v\,dt \qquad v = \int a\,dt$

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SECTION I

Answer all questions.

ALL working must be clearly shown.

1. (a) (i) Determine the remainder when $f(x) = ax^3 + 7x^2 - 7x - 3$ is divided by $x - 1$.

(2 marks)

(ii) If the remainder when $f(x)$ is divided by $(x + 3)$ is equal to the remainder determined in (a) (i), find the value of a.

(3 marks)

GO ON TO THE NEXT PAGE

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(b) Consider the function $g(x) = x^2 + (m + 4)x + 4m = 0$, which has real and equal roots. Use the discriminant of the given equation to determine the values for m. You may use the grid provided to assist you.

(5 marks)

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(c) Let $h(x) = 2x^2 + 8x - 10$.

(i) Express $h(x)$ in the form $a(x + b)^2 + c$.

(3 marks)

(ii) State the minimum value of $h(x)$.

(1 mark)

(iii) Determine the value of x for which $h(x)$ is a minimum.

(1 mark)

Total 15 marks

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2. (a) Given that $log_2 (6 + \sqrt{12}) - \log_2 (3 + \sqrt{a}) = \log 10$, find the value of a.

(5 marks)

(b) Determine the set of values of x for which $\frac{2 - x}{4x - 9} < 0$.

(4 marks)

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(c) Alice deposited $4000 into her new savings account at Bank of Fortune, which pays interest at 8% per annum. The bank's compounded interest is represented by the geometric progression

$A = P\left(1 + \frac{R}{100}\right)^T$ where A is the amount of money accumulated after T years,

R, the percentage rate of interest per annum and T, a positive integer, the time in years.

Determine the number of years it would take Alice's money to **at least triple**.

(6 marks)

Total 15 marks

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Section II

Answer ALL questions.

ALL working must be clearly shown.

3. (a) The coordinates for the centre of a circle is (2, 1) and the coordinates for a point on its circumference is (3, 3).

(i) Determine the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$.

(4 marks)

(ii) The circle intersects the x and y-axes at three points. Determine the coordinates of the three points of intersection.

(4 marks)

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(iii) Determine the equation of the tangent to the circle at the point (3, 3).

(4 marks)

(b) The position vectors of two points, P and Q, relative to a fixed origin, O, are given by $OP = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $OQ = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$. Determine the unit vector in the direction of $\boldsymbol{PQ}$, giving your answer in simplest surd form.

(4 marks)

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(c) Given that $\cos M = \frac{24}{25}$ and that angle M is acute, determine the value for $\tan 2M$.

(4 marks)

Total 20 marks

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SECTION III

Answer ALL questions.

ALL working must be clearly shown.

4. (a) (i) Differentiate $\sin x + \cos 4x$ with respect to x.

(2 marks)

(ii) Differentiate $\frac{2x^3 + 2}{2x + 1}$ with respect to x.

(3 marks)

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(b) Use the principles of differentiation to compute the stationary value of the function $y = x^2 - 4x + 2$.

(4 marks)

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(c) A motorist starts from a point *X* and travels 60 m due north to a point *Y* at a constant speed of 4 ms^{-1}. He stays at *Y* for 25 seconds and then travels at a constant speed of 10 ms^{-1} for 100 m due south to a point, *Z*. Calculate

(i) the average speed for the whole journey

(4 marks)

(ii) the average velocity of the whole journey.

(2 marks)

Total 15 marks

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5. (a) Determine the following integrals, giving each answer in its simplest form.

(i) $\int 2x^2 + 3x \, dx$

(2 marks)

(ii) $\int 2 \sin 3x \, dx$

(3 marks)

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(b) Using an integration method, calculate the area of the region in the first quadrant under the graph $y = 3 \sin x$.

(4 marks)

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(c) A particle starting from rest travels in a straight line with an acceleration, a, given by $a = t^2$ where t is the time in seconds.

(i) Determine the velocity, v, of the particle in terms of time, t.

(2 marks)

(ii) Calculate the displacement, s, of the particle in the interval of time $t = 0$ to $t = 2$.

(4 marks)

Total 15 marks

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SECTION IV

Answer ALL questions.

ALL working must be clearly shown.

6. (a) Two fair tetrahedral dice with faces numbered 1, 2, 3, 4 are rolled. The numbers obtained on the turned-down face of each dice are noted.

Create a sample space table listing ALL possible outcomes for the two dice.

(3 marks)

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(b) Using your sample space table created in (a), or otherwise, determine the probability of obtaining a 4

(i) on **both** dice

(1 mark)

(ii) on **at least one** dice

(1 mark)

(iii) on **exactly one** dice.

(1 mark)

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(iv) Show that obtaining a 4 on **both** dice are independent events.

(2 marks)

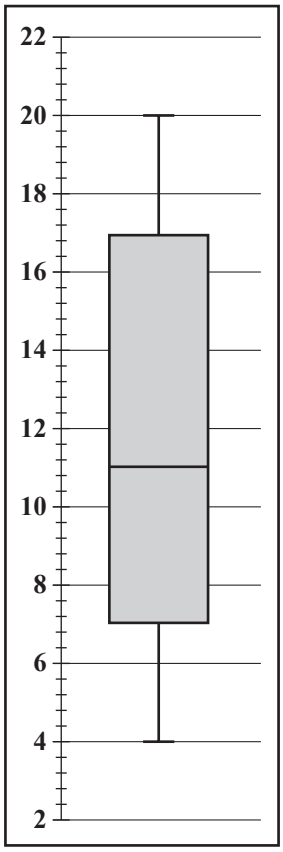
(v) Determine the probability of obtaining a 4 on **both** dice, given that a 4 was obtained on **at least one** dice.

(2 marks)

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(c) The scores of a class of 30 students on a Mathematics test were used to draw the box plot below. (The total score possible is 20 marks.)



Using the box plot, determine the following:

(i) The median score

(1 mark)

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(ii) The range of the scores

(1 mark)

(iii) The semi-interquartile range of the scores

(2 marks)

(iv) Comment on the shape of the distribution of the scores.

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(1 mark)

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A student wants to determine the mean score for the data set.

(v) State ONE reason why it would be impossible to determine the mean score from the box plot.

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(1 mark)

(vi) State what additional piece of information would be needed to determine the mean score.

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(1 mark)

(vii) Given that the sum of the 30 scores for the class is 354 and the sum of the squares of the scores is 4994, determine the standard deviation for the data set.

(3 marks)

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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