FORM TP 2013037

TEST CODE **01254020**

MAY/JUNE 2013

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

07 MAY 2013 (p.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. DO NOT open this examination paper until instructed to do so.
2. This paper consists of FOUR sections. Answer ALL questions in Section 1, Section 2 and Section 3.
3. Answer ONE question in Section 4.
4. Write your solutions with full working in the booklet provided.
5. A list of formulae is provided on page 2 of this booklet.

Required Examination Materials

Electronic calculator (non programmable)
Geometry Set
Mathematical Tables (provided)
Graph paper (provided)

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LIST OF FORMULAE

Arithmetic Series $T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric Series $T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r},\ -1 < r < 1 \text{ or } |r| < 1$

Circle $x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x+f)^2 + (y+g)^2 = r^2$

Vectors $\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)} \text{ where } \mathbf{v} = x\mathbf{i} + y\mathbf{j}$

Trigonometry $\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Differentiation $\frac{\mathrm{d}}{\mathrm{d}x}(ax+b)^n = an(ax+b)^{n-1}$

$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$

$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$

Statistics $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$

Probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Kinematics $v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$

GO ON TO THE NEXT PAGE

SECTION 1

Answer BOTH questions.

All working must be clearly shown.

1. (a) Let $f(x) = x^3 - x^2 - 14x + 24$.

(i) Use the factor theorem to show that $x + 4$ is a factor of $f(x)$. **(2 marks)**

(ii) Determine the other linear factors of $f(x)$. **(3 marks)**

(b) A function $f(x)$ is given by $f(x) = \dfrac{2x - 1}{x + 2}$.

(i) Find an expression for the inverse function $f^{-1}(x)$. **(3 marks)**

(ii) The function g is given by $g(x) = x + 1$. Write an expression for the composite function, $fg(x)$. Simplify your answer. **(2 marks)**

(c) Given that $5^{3x-2} = 7^{x+2}$, show that

$x = \dfrac{2(\log 5 + \log 7)}{(\log 125 - \log 7)}$. **(4 marks)**

Total 14 marks

2. (a) Let $f(x) = 3x^2 + 6x - 1$.

(i) Express $f(x)$ in the form $a(x + h)^2 + k$ where h and k are constants. **(3 marks)**

(ii) State the minimum value of $f(x)$. **(1 mark)**

(iii) Determine the value of x for which $f(x)$ is a minimum. **(1 mark)**

(b) Find the set of values of x for which $2x^2 + 3x - 5 \geq 0$. **(4 marks)**

(c) Find the sum to infinity of the following series:

$\dfrac{1}{4} + \dfrac{2}{4^2} + \dfrac{1}{4^3} + \dfrac{2}{4^4} + \ldots$

Note: This series can be rewritten as the sum of two geometric series. **(5 marks)**

Total 14 marks

SECTION 2

Answer BOTH questions.

All working must be clearly shown.

3. (a) (i) A circle, C, has centre with coordinates A (2,1) and passes through the point B (10,7).

Express the equation of the circle in the form $x^2 + y^2 + hx + gy + k = 0$, where h, g and k are integers to be determined. **(3 marks)**

(ii) The line l is a tangent to the circle C at the point B. Find an equation for l. **(3 marks)**

(b) The position vectors of two points, P and Q, relative to a fixed origin, O, are $10\mathbf{i} - 8\mathbf{j}$ and $\lambda\mathbf{i} + 10\mathbf{j}$ respectively, where λ is a constant.

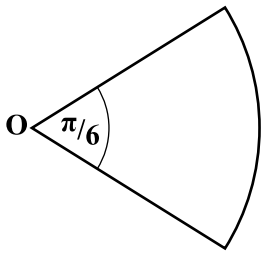
Find the value of λ such that $\overrightarrow{\mathbf{OP}}$ and $\overrightarrow{\mathbf{OQ}}$ are perpendicular. **(3 marks)**

(c) The position vectors of A and B with respect to a fixed origin, O, are given by $\overrightarrow{OA} = -2\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{OB} = 3\mathbf{i} - 7\mathbf{j}$ respectively.

Find the unit vector in the direction of AB. **(3 marks)**

Total 12 marks

4. (a)



The diagram shows a sector cut from a circle of centre O. The angle at O is $\frac{\pi}{6}$. If the perimeter of the sector is $\frac{5}{6}(12 + \pi)$ cm, what is its area? **(4 marks)**

(b) Solve the equation $2\cos^2\theta + 3\sin\theta = 0$ for $0 \leq \theta \leq 360°$. **(5 marks)**

(c) Given that $\tan(\theta - \alpha) = \frac{1}{2}$ and that $\tan\theta = 3$, use the appropriate compound angle formula to find the value of the acute angle α. **(3 marks)**

Total 12 marks

GO ON TO THE NEXT PAGE

SECTION 3

Answer BOTH questions.

All working must be clearly shown.

5. (a) Given that $y = x^3 - 3x^2 + 2$. Find

(i) the coordinates of the stationary points of y **(5 marks)**

(ii) the second derivative of y and hence determine the nature of EACH of the stationary points. **(5 marks)**

(b) Differentiate $y = (5x + 3)^3 \sin x$ with respect to x, simplifying your result as far as possible. **(4 marks)**

Total 14 marks

6. (a) Find $\int (5x^2 + 4)\, dx$. **(2 marks)**

(b) Evaluate $\int_0^{\pi/2} (3 \sin x - 5 \cos x)\, dx$. **(4 marks)**

(c) A curve passes through the points P(0, 8) and Q(4, 0) and is such that $\frac{dy}{dx} = 2 - 2x$.

Find the area of the finite region bounded by the curve in the first quadrant.

(8 marks)

Total 14 marks

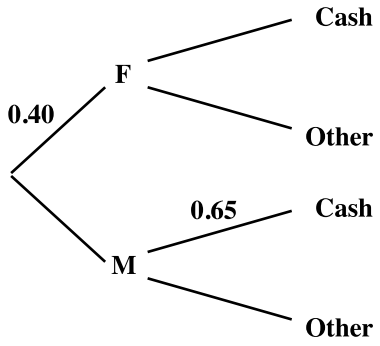
SECTION 4

Answer ONLY ONE question.

All working must be clearly shown.

7. (a) Of the persons buying petrol at a service station, 40 per cent are females. Of the females, 30 per cent pay for their petrol with cash, and of the males, 65 per cent pay for their petrol with cash.

(i) Copy and complete the following tree diagram, by putting in all the missing probabilities, to show this information. **(2 marks)**



(ii) What is the probability that a customer pays for petrol with cash? **(3 marks)**

(iii) Determine which is the more likely event:

Event T: Customer is female, GIVEN that the petrol is paid WITH cash.
Event V: A male customer does NOT pay for petrol with cash. **(4 marks)**

(b) The marks obtained by 30 students on an English exam are given as

58	92	41	89	72	66	51	63	80	40
69	45	83	76	53	56	75	50	99	50
85	63	58	75	66	56	81	74	51	94

(i) State ONE advantage of using a stem and leaf diagram versus a box and whiskers plot to display the data. **(1 mark)**

(ii) Construct a stem and leaf diagram to show the data. **(3 marks)**

(iii) Determine the median mark. **(2 marks)**

(iv) Calculate the semi inter-quartile range of the marks. **(3 marks)**

(v) Two students are chosen at random from the class.

Determine the probability that both scored less than 50 on the exam. **(2 marks)**

Total 20 marks

8. (a) A particle starts from rest and accelerates uniformly to 20 m s^{-1} in 5 seconds. It continues at this velocity for 10 seconds. It then accelerates again uniformly to a velocity of 60 m s^{-1} in 5 seconds. The particle then decelerates uniformly until it comes to rest, 15 seconds later.

(i) **On the graph paper provided**, draw a velocity-time graph to illustrate the motion of the particle. **(3 marks)**

(ii) From your graph determine

a) the total distance, in metres, travelled by the particle **(4 marks)**

b) the average velocity of the particle for the entire journey. **(2 marks)**

(b) A particle travels in a straight line in such a way that after t seconds its velocity, v, from a fixed point, O, is given by the function $v = 3t^2 - 18t + 15$.

Calculate

(i) the values of t when the particle is instantaneously at rest **(3 marks)**

(ii) the distance travelled by the particle between 1 second and 3 seconds **(3 marks)**

(iii) the value of $\frac{dv}{dt}$ when

a) $t = 2$ seconds **(2 marks)**

b) $t = 3$ seconds. **(1 mark)**

(iv) Give an interpretation for the value in

a) 8 (b) (iii) a) **(1 mark)**

b) 8 (b) (iii) b). **(1 mark)**

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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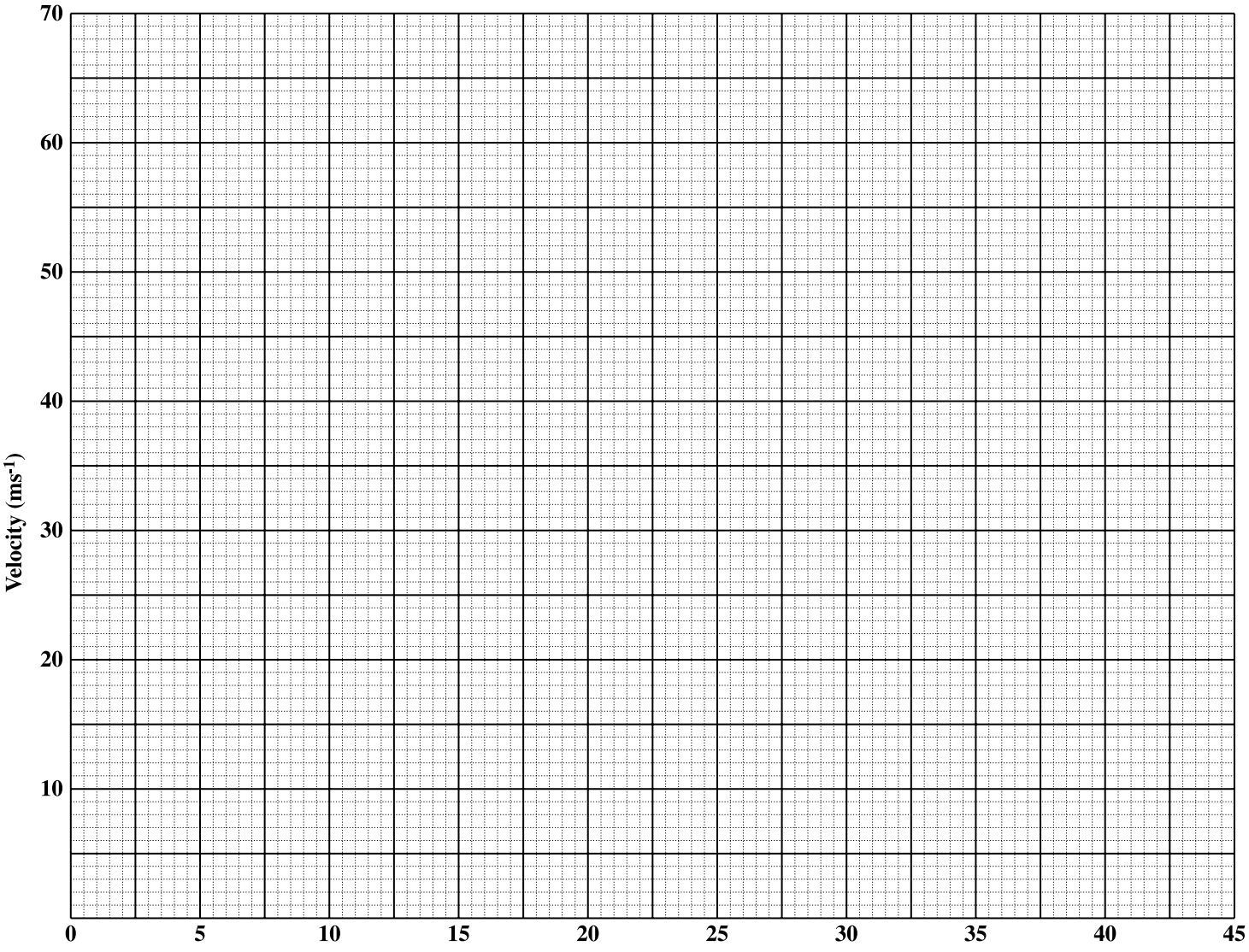
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Answer Sheet for Question 8 (a) **Candidate Number .....**



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