FORM TP 2023046

TEST CODE 01254020

MAY/JUNE 2023

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of SIX questions. Answer ALL questions.
2. Write your answers in the spaces provided in this booklet.
3. Do NOT write in the margins.
4. A list of formulae is provided on page 4 of this booklet.
5. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
6. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Required Examination Materials

Electronic calculator (non-programmable)
Geometry set

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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LIST OF FORMULAE

Arithmetic Series

$$T_n = a + (n-1)d \qquad S_2 = \frac{n}{2}[2a + (n-1)d]$$

Geometric Series

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r}, -1 < r < 1 \text{ or } |r| < 1$$

Circle

$$x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x+f)^2 + (y+g)^2 = r^2$$

Vectors

$$\mathbf{v} = \frac{\hat{\mathbf{v}}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{a}| \bullet |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)} \text{ where } \mathbf{v} = x\mathbf{i} + y\mathbf{j}$$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Kinematics

$$v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2 \qquad v = \frac{dx}{dt} = \dot{x}$$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \ddot{x}$$

Calculus

$$\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1} \qquad \frac{d}{dx}\sin ax = a\cos ax$$

$$\frac{d}{dx}\cos ax = -a\sin ax \qquad s = \int v\,dt \qquad v = \int a\,dt$$

Statistics

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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SECTION I

ALGEBRA, SEQUENCES AND SERIES

ALL working must be clearly shown.

1. (a) Solve the equation $3^{2x+1} - 5(3^x) - 2 = 0$, giving your answer to 3 decimal places.

(5 marks)

(b) (i) Given that $3x + 2$ is a factor of $3x^3 + bx^2 - 3x - 2$, find the value of b.

(3 marks)

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(ii) Hence, factorize completely $3x^3 + bx^2 - 3x - 2$.

(3 marks)

(c) Determine the value(s) of p for which the function $px^2 + 3x + 2p$ has two real distinct roots, giving your answer in its simplest form.

(4 marks)

Total 15 marks

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2. Given that $f(x) = 3x^2 - 9x + 1$,

(a) (i) express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are real numbers

(3 marks)

(ii) state the coordinates of the minimum point of $f(x)$.

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(2 marks)

(b) The equation $3x^2 - 6x - 2 = 0$ has roots α and β. Find the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$.

(4 marks)

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(c) John's grandparents started a university fund for him at a bank, with $4000. The bank offered two options for interest.

Option 1 — $240 per annum
Option 2 — 5% of the current balance per annum

Determine the sum of money in the university fund at the beginning of the ninth year for **both** options.

(6 marks)

Total 15 marks

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SECTION II

COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY

ALL working must be clearly shown.

3. (a) The equation of a circle is $x^2 + y^2 - 8x - 18y + 93 = 0$.

(i) Determine the coordinates of the centre of the circle.

(2 marks)

(ii) Find the length of the radius.

(1 mark)

(iii) Find the equation of the normal to the circle at the point (2, 8).

(3 marks)

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(b) The position vectors of two points, A and B, relative to an origin O, are such that $\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j}$ and $\overrightarrow{OB} = 5\mathbf{i} - 4\mathbf{j}$. Determine

(i) the unit vector $\boldsymbol{AB}$

(3 marks)

(ii) the acute angle AÔB, in degrees, to **1 decimal place.**

(3 marks)

(c) Solve the equation $2 \sin^2 \theta = 3 \cos \theta$ where $0° < \theta < 180°$.

(4 marks)

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(d) Prove the identity $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{2\tan x}{\cos x}$.

(4 marks)

Total 20 marks

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NOTHING HAS BEEN OMITTED.

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SECTION III

INTRODUCTORY CALCULUS

ALL working must be clearly shown.

4. (a) A function given by $y = ax^2 + bx + c$ has a gradient of $9 - \frac{1}{2}x$ at a stationary value of 5.

(i) Determine the values of a, b and c in the function.

(5 marks)

(ii) Determine the nature of the stationary point.

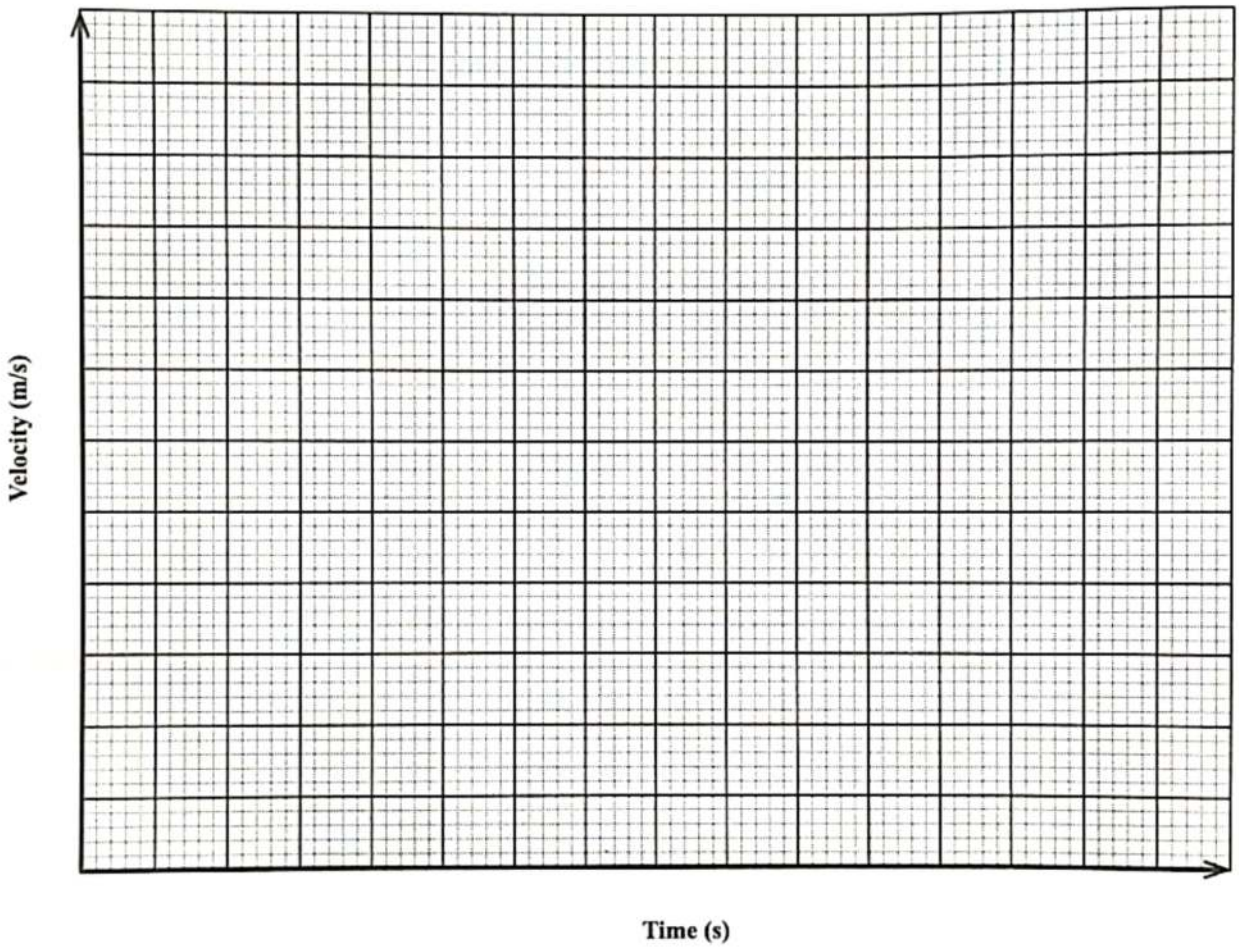
(2 marks)

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(b) A drone tracks the movement of an object in motion on the ground. The following movements are recorded.

- It moves at a constant velocity of 4 m/s for 5 seconds.
- Its velocity increases uniformly for 3 seconds to 10 m/s.
- It moves at that velocity for 7 seconds.
- It slows uniformly until it comes to rest after 4 seconds.

(i) Sketch the graph of the movement of the object.



(3 marks)

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(ii) Calculate the distance travelled by the object in the second part of the journey.

(3 marks)

(iii) Determine the object's acceleration in the final part of the journey.

(2 marks)

Total 15 marks

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5. (a) Consider a toy truck moving along a miniaturized racing course with an acceleration of $a(t) = 9t^2 + 2t - 1$, where t is the time in seconds. Assume that the speed of the toy truck is measured in cm/second.

(i) Outline how an equation for the speed of the truck would be found, using the given equation for the acceleration.

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(1 mark)

(ii) Find the speed of the truck at $t = 10$ seconds.

(3 marks)

(iii) Find the distance covered by the truck between 5 and 10 seconds. Express your answer to **the nearest whole number**.

(4 marks)

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(b) Find

(i) $\int (2x+3)^2 \, dx$

(2 marks)

(ii) $\int 5 \cos 2x \, dx.$

(2 marks)

(c) If $\frac{dy}{dx} = 6x - 10$ and $y = 12$ when $x = 0$, find the equation for y in terms of x.

(3 marks)

Total 15 marks

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SECTION IV

PROBABILITY AND STATISTICS

ALL working must be clearly shown.

6. The following stem and leaf diagram represents the scores, out of 80, of students in an Additional Mathematics exam.

3	0 3 7
4	2 4 6 7 9
5	1 3 3 6
6	0 7 7
7	1 9

(a) Write the raw data set that was used to construct the diagram above.

(2 marks)

(b) Determine the following measures for the data set.

(i) The median exam score

(2 marks)

(ii) The mean exam score

(2 marks)

(iii) The modal score(s)

(2 marks)

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(iv) Given that for the data set $\sum x^2 = 48999$ and $\sum x = 885$, find the standard deviation of the data set using the formula

$$S = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}.$$

(4 marks)

(c) If a student needed to score at least half the total marks possible to pass the exam, determine the probability of a student failing the exam. Give your answer to **2 decimal places**.

(2 marks)

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(d) Given that a randomly selected student has passed the exam, what is the probability that the student scored over 60? Give your answer to **2 decimal places**.

(2 marks)

(e) Based on the given stem and leaf diagram, describe the distribution of the data set.

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(2 marks)

(f) Your friend suggested that a bar graph or histogram could be used to represent the data. Advise your friend on which graph is the better option giving ONE reason to support your answer.

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(2 marks)

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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