FORM TP 2014038

TEST CODE 01254032

MAY/JUNE 2014

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION

ADDITIONAL MATHEMATICS

Paper 032 – General Proficiency

ALTERNATIVE

1 hour 30 minutes

09 JUNE 2014 (p.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of ONE question. Answer the given question.
2. Write your solutions with full working in the booklet provided.
3. A list of formulae is provided on page 2 of this booklet.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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LIST OF FORMULAE

Arithmetic Series $T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric Series $T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1 - r},\ -1 < r < 1 \text{ or } |r| < 1$

Circle $x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x + f)^2 + (y + g)^2 = r^2$

Vectors $\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)} \text{ where } \mathbf{v} = x\mathbf{i} + y\mathbf{j}$

Trigonometry $\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Differentiation $\frac{\mathrm{d}}{\mathrm{d}x}(ax + b)^n = an(ax + b)^{n-1}$

$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$

$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$

Statistics $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$

Probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Kinematics $v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$

GO ON TO THE NEXT PAGE

1. **(a)** **A student has to compute the area under the graph of a function. He reasons that he can do** so by subdividing the area into an infinitely large number of rectangles. To help himself, he investigates by finding the area under the graph of the function $f(x) = x$ over the interval [0,1], using the method of circumscribed rectangles as shown in Figure 1.

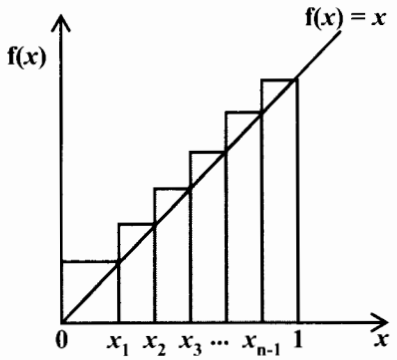


Figure 1. Circumscribed Rectangles

(i) The student subdivides the interval [0, 1] into n equal subintervals. Calculate the width, Δx, of each subinterval. **(1 mark)**

(ii) Let the points of subdivision be $x_0 = 0, x_1, x_2, x_3, ..., x_{n-1}, x_n = 1$ as shown in Figure 1.

Find the values of $x_1, x_2, x_3, ..., x_{n-1}$ in terms of n. **(1 mark)**

(iii) Determine the heights $h_1, h_2, h_3, ..., h_n$ of the circumscribed rectangles over each of the respective n subintervals. **(2 marks)**

(iv) Determine the area $A_1, A_2, A_3, ..., A_n$ of the respective circumscribed rectangles. **(2 marks)**

(v) Show that the sum, S_n, of the areas of these circumscribed rectangles is given by

$$S_n = \frac{n+1}{2n}.$$ **(3 marks)**

(Hint: You will need to evaluate the sum of a series. State any theorem used.)

(vi) a) Compute $S(n)$ for $n = 10, 20, 50$ and 100, giving your answers to three decimal places. **(2 marks)**

b) What number does $S(n)$ approach as n gets larger? **(1 mark)**

(b) The variables x and y are related by a law of the form $y = ax^n$, where a and n are integers. The approximate values for y, corresponding to the given values of x are shown in Table 1.

Table 1

x	2	3	4	5	6	7
y	50	250	775	1875	3900	7200

(i) Use logarithms to reduce this relation to a linear form, giving your values of lg x and lg y correct to two decimal places where appropriate. **(2 marks)**

(ii) **Using the graph paper provided** and a scale of 2 cm to represent 0.1 units on the x-axis, and 1 cm to represent 0.2 units on the y-axis, plot a suitable straight line graph of lg y against lg x. **(2 marks)**

(iii) Use your straight line graph to estimate the value of the constant a and the value of the constant n. **(4 marks)**

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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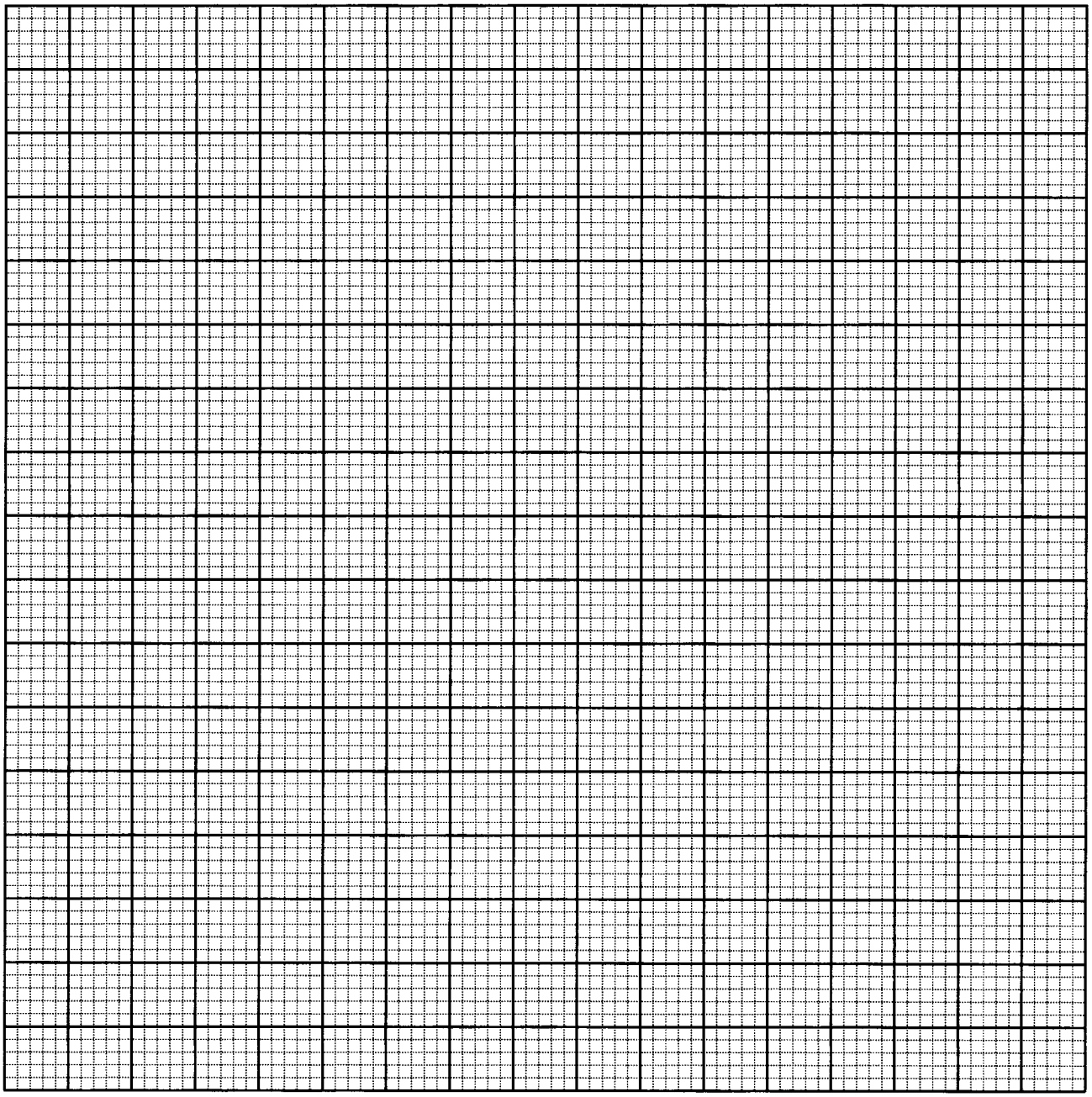
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Graph Sheet for Question 1 (b) (ii) **Candidate Number .....**



ATTACH THIS ANSWER SHEET TO YOUR ANSWER BOOKLET