FORM TP 2017037

TEST CODE **01254020**

MAY/JUNE 2017

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of FOUR sections. Answer ALL questions in Section I, Section II and Section III.
2. Answer ONE question in Section IV.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. A list of formulae is provided on page 4 of this booklet.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Required Examination Materials

Electronic calculator (non-programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

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LIST OF FORMULAE

Arithmetic Series $T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric Series $T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r},\ -1 < r < 1 \text{ or } |r| < 1$

Circle $x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x+f)^2 + (y+g)^2 = r^2$

Vectors $\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)} \text{ where } \mathbf{v} = x\mathbf{i} + y\mathbf{j}$

Trigonometry $\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Differentiation $\frac{\mathrm{d}}{\mathrm{d}x}(ax+b)^n = an(ax+b)^{n-1}$

$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$

$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$

Statistics $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$

Probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Kinematics $v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$

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SECTION I

Answer BOTH questions.

ALL working must be clearly shown.

1. (a) The function f is defined by

$$f(x) = \frac{2x + p}{x - 1}, \; x \neq 1 \text{ and } p \text{ is a constant.}$$

(i) Determine the inverse of $f(x)$.

(2 marks)

(ii) If $f^{-1}(8) = 5$, find the value of p.

(1 mark)

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(b) Given that the remainder when $f(x) = x^3 - x^2 - ax + b$ is divided by $x + 1$ is 6, and that $x - 2$ is a factor, determine the values of a and b.

(4 marks)

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(c) The values of the variables P and x in Table 1 obtained from an experiment are thought to obey a law of the form $P = Ax^{-k}$.

TABLE 1

x	1.58	2.51	3.98	6.30	10.0
P	121.5	110.6	106.2	99.1	93.8

(i) Use logarithms to reduce the equation to linear form.

(1 mark)

(ii) Using a suitable scale, plot the best fit line of the equation in (c) (i) on the graph paper **provided on page 9**. Use the space below to show your working.

(3 marks)

(iii) Hence, estimate the constants A and k.

(3 marks)

Total 14 marks

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2. (a) The quadratic equation $2x^2 + 6x + 7 = 0$ has roots α and β.

Calculate the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

(4 marks)

(b) Determine the range of values of x for which $\frac{2x + 3}{x + 1} \geq 0$.

(4 marks)

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(c) An accountant is offered a five-year contract with an annual increase. The accountant earned a salary of $53 982.80 and $60 598.89 in the third and fifth years respectively. If the increase follows a geometric series, calculate

(i) the amount paid in the first year

(4 marks)

(ii) the TOTAL salary earned at the end of the contract.

(2 marks)

Total 14 marks

SECTION II

Answer BOTH questions.

ALL working must be clearly shown.

3. (a) A circle C has an equation $x^2 + y^2 + 4x - 2y - 20 = 0$.

(i) Express the equation in the form $(x + f)^2 + (y + g)^2 = r^2$.

(2 marks)

(ii) State the coordinates of the centre and the value of the radius of circle C.

(2 marks)

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(iii) Determine the points of intersection of circle C and the equation $y = 4 - x$.

(4 marks)

(b) Given that $\mathbf{p} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{q} = \mathbf{i} + 5\mathbf{j}$, determine

(i) the product of the two vectors, **p** and **q**

(1 mark)

(ii) the angle between the two vectors.

(3 marks)

Total 12 marks

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4. (a) Figure 1 shows a plot of land, ABCD **(not drawn to scale)**. Section ABC is used for building and the remainder for farming. The radius BC is 10 m and angle BCD is a right angle.

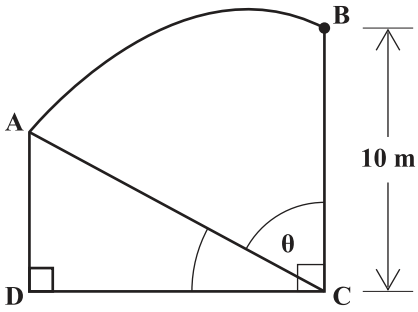


Figure 1

(i) If the building space is $\frac{50\pi}{3}$ m^2, calculate the angle ACB in radians.

(1 mark)

(ii) Working in radians, calculate the area used for farming.

(4 marks)

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(b) Given that

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2},$$

$$\cos \frac{\pi}{3} = \frac{1}{2} \text{ and}$$

$$\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2},$$

show **without using a calculator** that

$$\frac{\cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right)}{\sin \frac{2\pi}{3}} = \frac{\sqrt{2} + \sqrt{6}}{2\sqrt{3}}.$$

(4 marks)

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(c) Prove the identity

$$1 - \frac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta.$$

(3 marks)

Total 12 marks

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SECTION III

Answer BOTH questions.

ALL working must be clearly shown.

5. (a) Differentiate the expression $(1 + 2x)^3 (x + 3)$ with respect to x, simplifying your answer.

(4 marks)

(b) The point P $(-2, 0)$ lies on the curve $y = 3x^3 + 2x^2 - 24x$. Determine the equation of the normal to the curve at point P.

(5 marks)

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(c) Water is poured into a cylindrical container of radius 15 cm. The height of the water increases at a rate of 2 cms^{-1}. Given that the formula for the volume of a cylinder is $\pi r^2 h$, determine the rate of increase of the volume of water in the container in terms of π.

(5 marks)

Total 14 marks

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6. (a) Show that $\int_0^{\frac{\pi}{4}} (\sin x + 4 \cos x)\, dx = \frac{3\sqrt{2} + 2}{2}$.

(4 marks)

(b) Determine the equation of a curve whose gradient function $\frac{dy}{dx} = x + 2$, and which passes through the point P (2, 3).

(3 marks)

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(c) Evaluate $\int_1^2 (4 - x)^2 \, dx$.

(3 marks)

(d) Calculate the volume of the solid formed when the area enclosed by the straight line $y = \frac{x}{2}$ and the x-axis for $x = 0$ to $x = 6$ is rotated through 2π about the x-axis.

(4 marks)

Total 14 marks

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SECTION IV

Answer only ONE question.

ALL working must be clearly shown.

7. (a) The probability of a final-year college student receiving a reply for an internship programme from three accounting firms, *Q*, *R* and *S*, is 0.55, 0.25 and 0.20 respectively. The probability that the student receives a reply from firm *Q* and is accepted is 0.95. The probability that a student receives a reply from firms *R* and *S* and is accepted is 0.30 for each of them.

(i) Draw a tree diagram to illustrate the information above.

(4 marks)

(ii) Determine the probability that the student will be accepted for an internship programme.

(4 marks)

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(b) Table 2 shows the length, in cm, of 20 spindles prepared by a carpenter to build a railing for an existing staircase.

TABLE 2

1.5	3.2	6.1	9.4	11.0	12.6	17.0	18.5	20.2	24.4
25.2	25.2	28.3	28.8	29.1	30.4	32.5	34.6	38.3	38.4

Determine

(i) the mean length

(2 marks)

(ii) the modal length

(1 mark)

(iii) the median length

(1 mark)

(iv) the interquartile range for the data.

(2 marks)

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(c) A school cafeteria sells 20 chicken patties, 10 lentil patties and 25 saltfish patties daily. On a particular day, the first student ordered 2 patties but did not specify the type. The vendor randomly selects 2 patties.

(i) Calculate the probability that the first patty selected was saltfish.

(4 marks)

(ii) Given that the first patty was saltfish, calculate the probability that the second patty was NOT saltfish.

(2 marks)

Total 20 marks

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8. (a) The displacement, s, of a particle from a fixed point, O, is given by $s = t^3 - \frac{5}{2}t^2 - 2t$ metres at time, t seconds.

(i) Determine the velocity of the particle at $t = 3.5$ s, clearly stating the correct unit.

(3 marks)

(ii) If the particle is momentarily at rest, find the time, t, at this position.

(3 marks)

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(b) A vehicle accelerates uniformly from rest for 75 m then travels for another 120 m at its maximum speed. The vehicle later stops at a traffic light. The distance from rest to the traffic light is 240 m and the time for the journey is 15 seconds.

(i) In the space below, sketch a velocity–time graph to illustrate the motion of the vehicle.

(3 marks)

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(ii) Calculate the length of time the vehicle maintains constant speed.

(7 marks)

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(iii) Calculate the maximum velocity attained.

(2 marks)

(iv) Determine the acceleration of the vehicle.

(2 marks)

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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